Q12 - Übung zu Stammfunktionen

Aufgabe 1:

a) $f(x) = -5x^2 \;;\; F(x) = -\frac{5}{3}x^3$

b) $f(x) = -2\cos x \;;\; F(x) = -2\sin x$

c) $f(x) = 5 - e^x \;;\; F(x) = 5x - e^x$

d) $f(x) = (x-2)\cdot x^2 = x^3 - 2x^2 \;;\; F(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3$

e) $f(x) = \frac{3}{x^2} = 3\cdot x^{-2} \;;\; F(x) = 3\cdot\frac{1}{-1}\cdot x^{-1} = -3\cdot x^{-1}$

f) $f(x) = \frac{x^3}{4} - \frac{2}{x^2} = \frac{1}{4}x^3 - 2\cdot x^{-2} \;;\; F(x) = \frac{1}{16}x^4 + 2\cdot x^{-1}$

g) $f(x) = 2x\cdot e^{x^2} \;;\; F(x) = e^{x^2}$ (Umkehrung der Kettenregel)

h) $f(x) = 2\cdot e^{-x} \;;\; F(x) = -2\cdot e^{-x}$

Aufgabe 2:

a) $f(x) = \frac{2}{3}x^2 + 2 \;;\; F_c(x) = \frac{2}{9}x^3 + 2x + c$ mit $F_c(3) = 7$

$\frac{2}{9}\cdot 27 + 6 + c = 7 \Rightarrow c = -5$

$\Rightarrow F_{-5}(x) = \frac{2}{9}x^3 + 2x - 5$

b) $f(x) = 2x^3 - x + 1 \;;\; F_c(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 + x + c$ mit $F_c(2) = 4$

$\frac{1}{2}\cdot 16 - \frac{1}{2}\cdot 4 + 2 + c = 4 \Rightarrow c = -4$

$\Rightarrow F_{-4}(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 + x - 4$

c) $f(x) = \sin x - 2x \;;\; F_c(x) = -\cos x - x^2 + c$ mit $F_c(0|2)$

$-\underbrace{\cos 0}_{1} - 0 + c = 2 \Rightarrow c = 3$

$\Rightarrow F_3(x) = -\cos x - x^2 + 3$

d) $f(x) = 2 - 2e^x \;;\; F_c(x) = 2x - 2e^x + c$ mit $F_c(1) = e$

$2 - 2e + c = e \Rightarrow c = 3e - 2$

$\Rightarrow F_{3e-2}(x) = 2x - 2e^x + 3e - 2$

Aufgabe 3:

a) $F'(x) = x^3 - 3x^2 + 2 = f(x)$

b) $F'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x = f(x)$

c) $F'(x) = e^{2x} \cdot 2 \cdot (x-1) + e^{2x} \cdot 1 = e^{2x} \cdot (2x - 2 + 1) = e^{2x} \cdot (2x-1) = f(x)$

d) $F'(x) = \frac{2 \cdot (x+1) \cdot 1}{(x+1)^2} - \frac{(x+1) \cdot 0 - 2 \cdot 1}{(x+1)^2} = \frac{2x+2}{(x+1)^2} - \frac{-2}{(x+1)^2} = \frac{2x+4}{(x+1)^2} = f(x)$

Aufgabe 4:

Geg.: Nst. $x_1 = -3$; $x_2 = -1$; $x_3 = 3$; $P(2|-10) \in G_f$

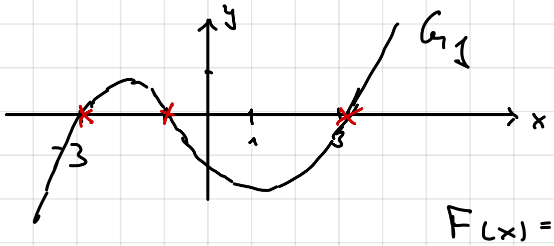
a) Nullstellenform: $f(x) = a \cdot (x+3) \cdot (x+1) \cdot (x-3)$

$= a \cdot (x^2 - 9)(x+1) = a \cdot (x^3 + x^2 - 9x - 9)$

$P \in G_f \Rightarrow a \cdot (4-9) \cdot 3 = -10 \Rightarrow a = \frac{2}{3}$

$\Rightarrow f(x) = \frac{2}{3}(x^3 + x^2 - 9x - 9) = \frac{2}{3}x^3 + \frac{2}{3}x^2 - 6x - 6$

Skizze:



$F(x) = \frac{2}{12}x^4 + \frac{2}{9}x^3 - 3x^2 - 6x$

b) $\int_{-5}^{5} f(x)\,dx = \left[\frac{2}{12}x^4 + \frac{2}{9}x^3 - 3x^2 - 6x\right]_{-5}^{5}$

$= \left[\cancel{\frac{625}{12}} + \frac{250}{9} - \cancel{75} - 30\right] - \left[\cancel{\frac{625}{12}} - \frac{250}{9} - \cancel{75} + 30\right]$

$= \frac{500}{9} - 60 = -\frac{40}{9} = -4\frac{4}{9}$

$\Rightarrow$ Die Fläche unter dem Graphen der Funktion überwiegt!

c) $A = \left|\int_{-5}^{-3} f(x)\,dx\right| + \int_{-3}^{-1} f(x)\,dx + \left|\int_{-1}^{3} f(x)\,dx\right| + \int_{3}^{5} f(x)\,dx$

$|F(-3) - F(-5)| + (F(-1) - F(-3) + |F(3) - F(-1)| + (F(5) - F(3)) =$

$\left|-\frac{3}{2} - \frac{565}{18}\right| + \left(\frac{53}{18} + \frac{3}{2}\right) + \left|-\frac{51}{2} - \frac{53}{18}\right| + \left(\frac{485}{18} + \frac{51}{2}\right)$

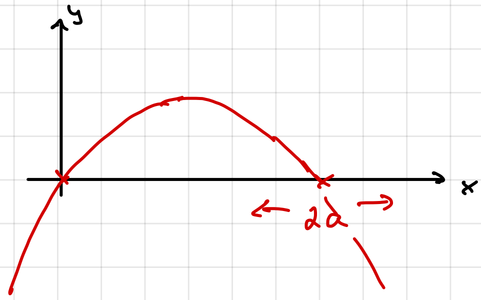
$\frac{296}{9} + \frac{40}{9} + \frac{256}{9} + \frac{472}{9} = \frac{1064}{9}$

$= 118\frac{2}{9}$

Aufgabe 5: $f_a(x) = -x^2 + 2ax$ mit $a \in \mathbb{R}^+$

a) <u>Nst:</u> $f_a(x) = x \cdot (-x + 2a) = 0 \Rightarrow x_1 = 0 \,;\, x_2 = 2a$

Skizze:



b) $\int_0^{2a} f(x)\,dx = \int_0^{2a} (-x^2 + 2ax)\,dx = \left[-\frac{1}{3}x^3 + ax^2 \right]_0^{2a} \overset{!}{=} 36$

$\Rightarrow \left(-\frac{1}{3}\cdot 8a^3 + a \cdot 4a^2\right) - \left(-\frac{1}{3}\cdot 0^3 + a \cdot 0^2\right) = 36$

$-\frac{8}{3}a^3 + \frac{12}{3}a^3 = 36$

$\frac{4}{3}a^3 = 36$

$a^3 = 27 \Rightarrow \boxed{a = 3}$